A New Perspective on Time and Physical Laws

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Craig Callender claims that 'time is the great informer', meaning that the directions in which our 'best' physical theories inform are temporal. This is intended to be a metaphysical claim, and as such expresses a relationship between the physical world and information-gathering systems such as ourselves. This article gives two counterexamples to this claim, illustrating the fact that time and informative strength doubly dissociate, so the claim cannot be about physical theories in general. The first is a case where physical theories inform in directions that we have no reason to regard as temporal. The second is a case where our best physical theories fail to inform in directions that we have independent (pre-theoretic) reasons to regard as temporal. Taking these two cases into account suggests that the connection Callender makes between time and informativeness is perspectival. The second case demonstrates that although scientists often seek information in temporal directions, the behaviour of the physical world can present serious difficulties for finding it. In response, this article proposes a perspectival reading of Callender's claim, according to which the connection between time and informative strength has more to do with the aims and objectives of science than the workings of the physical world.

1. Introduction

We begin with the problem of how to distinguish time from space in post-relativistic physics. It will be clear to many readers that time and space are essentially different, at least in the way we experience them. Getting lost, for example, is a rather different problem from being late; travel is permitted in any direction of space, but the same cannot be said for time; and spatially distant objects can be seen or accessed in a way that temporally distant objects cannot. These (and other) differences are encoded in a host of ways across physics. Despite relativistic physics teaching us about some important inter-dependence relationships between space and time, their differences are nonetheless encoded by asymmetries in the models of the theory. There remain questions about whether there might be a philosophical underpinning of the various

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asymmetries between time and space that we find in physical theories, and about whether such asymmetries are physically necessary.[1]

Callender ([2017], chap. 7) addresses these questions with his attempt to 'bind together' the differences between time and space, by claiming that 'time is the great informer', meaning that the direction in which the laws of our 'best' theories inform is temporal, not spatial.[2] This article gives two counterexamples to this claim. The first is a case where physical theories inform in directions that we have no reason to regard as temporal. The second is a case where our best physical theories fail to give precise information over short timescales or accurate information over long timescales, where we have independent (pre-theoretic) reasons to regard these directions as temporal. Taking these two cases into account breaks the connection Callender makes between time and the informative strength of physical laws. The second case demonstrates that although scientists often seek information in temporal directions, the behaviour of the physical world can present serious difficulties for finding it. This motivates the view that the connection between time and informative strength has more to do with the aims and objectives of science than the workings of the physical world.

The positive proposal of this article, namely, that time is connected to the informative strength of physical laws only for a certain set of scientific aims, is related to 'temporal perspectivalism', a position advanced in (Baron and Evans [2021]). Their reading of Callender's argument leads to the conclusion that the asymmetry between time and space is an artefact of our human epistemic situation. The alternative interpretation offered in the present work is also a form of perspectivalism, but the scope is narrower: it is only the connection Callender makes between time and informative strength, and not the asymmetry between time and space per se, that is reduced to a set of perspectives. However, the perspectivalism in this case is more worrying for objectivity, because the set of perspectives in question is identified by the specific aims of some human scientists, rather than the epistemic situation of the entire population. The proposal is rooted in empiricism (which Callender claims also to be committed to), a weak version of which can be spelt out as follows: the informativeness of physical laws is contingent on their formal characteristics being a good representation of empirically accessible phenomena.

The structure of this article is as follows: Section 2 gives a summary of the main arguments including the 'perspectival' reading of Callender's argument offered in

[1] To be clear, this article is about asymmetries between time and space, and not about the asymmetry of time. It has nothing to do with differences between past and future, for example, and the thermodynamic arrow of time will not be mentioned. It is about differences between time and space, and to the extent that spacetime can be thought of as a single structure it is appropriate to refer to the 'asymmetry' of this structure as encoding some of these differences.

[2] A different response might consider this asymmetry to be a defining characteristic of any structure that can properly be called spatiotemporal. A proposal of this nature has been made use of in (Le Bihan and Linnemann [2019]), in order to strengthen connections between the asymmetric spacetime of general relativity and asymmetric 'quasi-spacetime' structures in theories of quantum gravity.

(Baron and Evans [2021]). Section 3 sets out the technical material necessary for understanding Callender's thesis and the two additional cases. Section 4 argues that the two additional cases are indeed counterexamples to Callender's claim, and explains how they motivate the alternative proposal connecting time to the aims of many scientific endeavours to provide certain forms of information. Section 4.3 compares the two forms of perspectivalism, gesturing towards some possible implications of their combination. Finally, section 5 briefly concludes.

2. Overview

Section 2.1 summarises the connection Callender makes between time and informativeness in physical laws. This is followed by an overview, in section 2.2, of the reading of his arguments suggested by Baron and Evans, leading to their proposal for a perspectival take on the time-space asymmetry. It then outlines some further discussion of Callender's argument in section 2.3, laying the groundwork for understanding the significance of the two counterexamples. Finally, in section 2.3, it outlines an alternative proposal: the connection between time and informativeness is an artefact of the aims and interests of scientists working on specific research programs. This alternative proposal can be viewed as a perspectival reading of Callender's argument, but where the perspectivalism applies specifically to the connection between time and informativeness rather than the asymmetry between time and space itself. This reading is, however, more problematic for the generality of Callender's metaphysical thesis than that suggested by Baron and Evans, because the perspectives to which his claim is reduced form a subset of the regimes used in scientific practice rather than the epistemic perspective of human beings in general.

2.1. Time as the great informer

Callender identifies features of time as it appears in physical theories, which distinguish it from space. He attempts to weave these together into a general metaphysical thesis by connecting to a Lewisian best systems account (BSA) of physical modality.[3] His strategy is to use the principles of the BSA to provide criteria for identifying a representative set of physical laws, whose mathematical form he analyses in order to give a general message about spatiotemporal structure. His conclusion is that 'time is the great informer'—time is the set of directions on a four-dimensional manifold of events in which laws give the most information. He summarises this view with the statement that 'time is that direction on the manifold of events in which we can tell the strongest or most informative stories' (Callender [2017], p. 142). So, the most informative directions on the manifold are labelled as 'time'. We begin

[3] See (Lewis [1983]) for details of this account. See also (Loewer [1996]; Maudlin [2007]; Cohen and Callender [2009]; Belot [2011]; Massimi [2018a]) (to name but a few) for discussion.

with the notion of informativeness and end up with a set of privileged 'time-like' directions. I call this hypothesis Callender's temporal informativeness proposal (TIP).

Callender ([2017], p. 120) gives several slight variants of this overall conclusion. He writes, 'the temporal direction is that direction on the manifold of events in which our best theories can tell the strongest, most informative "stories." Put another way, time is that direction in which our theories can obtain as much determinism as possible'. A slightly more precise formulation is given a little later: 'A *temporal direction* at a point p on $\langle M^d, g\rangle$ is that direction $(n, -n)$, where $(n, -n)$ is an unordered pair of nowhere vanishing vectors, in which our best theory tells the strongest, i.e., most informative, "story"'. His strategy does not 'assume that the "timelike" directions of g, if any, are themselves temporal directions. That's something we hope emerges from the analysis, not something put in' (pp. 142–43).

By way of explanation for his proposed conclusion, Callender ([2017], p. 120) states that 'strength is linked to time because it is deeply connected to the other temporal features of our universe. If I am right, "strength" is the glue that binds together many otherwise detachable features central to time'. This article does not aim to outright deny that there is any link at all between strength (informativeness) and time, but instead to propose a different, perspectival, explanation for it. It does so by acknowledging the lack of generality of the link Callender makes between strength (informativeness) and time, questioning what it is that unites the examples in physics where the link obtains and comparing them to examples where this link does not obtain. Callender's claim is that his 'attractive idea about time is more or less implied by a "systems" approach to laws'. He states also that 'the difference [between time and space, based on the way laws inform] ultimately lay in the distribution of physical properties' (p. 156). This part of his thesis will also be challenged, especially through the analysis of chaotic systems in section 3.3.

The argument for TIP is explicitly based on a Humean view of laws, according to which 'laws are simply the best summary of the facts' (Callender [2017], p. 140). This is where a commitment to Humean metaphysics and empiricist epistemology is made. According to Callender (p. 140), 'Humean theories seek to explain the laws *given* the distribution of actual facts', the actual facts being facts that are (at least in principle) empirically accessible.[4] In the language of Lewis ([1983]), the laws supervene on the mosaic of particular facts, meaning that there could not be a change in the laws without there being a corresponding change in the particular facts they describe (Bennett and McLaughlin [2018]). There are formally different ways to describe the same mosaic of facts, but these descriptions will not differ in content. The BSA provides a way of distinguishing between these formally different descriptions, elevating to the status of law parts of those descriptions, taking the form of

[4] Taking the distribution of empirical facts as given follows from the commitment to empiricism, central to Humean theories in general.

deductive systems, which satisfy some set of 'theoretical virtues', in particular simplicity and strength.[5] Callender invites us to 'consider various deductive systems, each of which only makes true claims about what exists' ([2017], p. 140). We then employ the theoretical virtues of strength and simplicity, emphasizing strength in particular, to identify the systems and associated laws that give the best description of the world. 'The motivation for the [BSA] theory is', according to Callender (p. 140), 'the idea that physical laws seek to describe accurately as much of the world as possible in a compact way'.[6] TIP comes out of an analysis of laws that, Callender claims, achieve this goal.

We need not concern ourselves very much with the notion of simplicity. Callender briefly discusses the simplifying role that time plays in physical laws, but asserts that 'there is nothing special about time here. [. . .] Space, for instance, is also the great simplifier'. For this reason, we must focus also on the virtue of strength, thought of as informativeness. 'In balancing simplicity and strength, a best system will [. . .] contain a way to generate some pieces of the domain of events given other pieces. In other words, it will favour algorithms, and short ones at that. The more of what happens that is generated by small input the better' (Callender [2017], p. 141). The claim is that a privileged time parameter is what gives physical laws their informative strength.

There are two versions of this argument given, an 'informal' one in (Callender [2017]) chapter 7 and a 'formal' one in chapter 8. It is indicated that these two versions of the argument are supposed to work together, that the latter version is intended to flesh out the former, adding credibility to it with the investigation into formal characteristics of equations used in physics. This version of his argument in chapter 8 proceeds by identifying a broad set of physical laws, generalized by their mathematical form, which provide algorithms that are informative in this sense. These laws take the form of partial differential equations (PDEs) that support well posed Cauchy problems. He investigates the mathematical form of these PDEs, and it is in this mathematical form that we find a privileged set of 'informative' directions that are defined as 'time-like': these equations use antecedent data to provide maximal information in time-like directions.

PDEs that support Cauchy problems do provide concise algorithms and are particularly informative over their domains of applicability. Section 4 explores Callender's justification for singling out these laws as being maximally informative, regardless of applicability. As described in section 3, it is their geometric structure—specifically, they are hyperbolic—that makes the distinction between time-like and space-like directions. Hyperbolic PDEs, when defined on some manifold, divide the tangent

5 Theoretical virtues may also include uniformity, elegance, generality and perhaps more, but we follow Callender in focusing only on simplicity and strength.

6 Seeking to describe the world is another expression of the empiricist commitments of Humean theories, if 'the world' is taken to refer to that which is observable, measurable, or otherwise empirically accessible. If 'the world' consisted of more than this it would be difficult to understand laws as being descriptive, because we would have no way of comparing descriptions with what they describe.

space of each point of that manifold into three regions by a pair of intersecting cone-shaped surfaces, in a manner analogous to the light-cone structure of special relativity and locally of general relativity. Restricting to well-posed problems means that antecedent data can only be defined on surfaces corresponding to space-like regions, and the PDEs evolve this in directions that correspond to the time-like. Therefore, the physical laws selected using the BSA criteria distinguish time from space, where the time-like and not the space-like directions are informative.

Callender suggests two alternative ways to read his proposal: one ‘conservative’ and one ‘radical’. So far we have been vague about what exactly the mosaic of facts or manifold of events or supervenience basis consists of. The two readings of the argument, to some degree, take care of this issue. According to the more conservative reading, the supervenience basis consists of events on a Lorentzian manifold endowed with a spacetime metric, which has an asymmetry between time and space built into its signature. The best systems merely embody the structure already assumed to be present in the manifold; the distinction between time and space does not emerge through systematization, but the connection between time and informativeness does. According to Callender ([2017], p. 151), however, the ‘radical perspective is the more natural development of the theory’. This second reading assumes only ‘the events *e* on *M*’ and the best systems ‘arrive at the spacetime metric *g* and laws *L* together’ (p. 150). So, Callender’s preferred interpretation of TIP means that by focusing on informative laws, the asymmetry between time and space is derived through systematization, and time is defined as the set of directions that are maximally informative.

2.2. Temporal perspectivalism

Baron and Evans ([2021]) explore some consequences of adopting the radical reading of TIP, according to which, ‘the *metrical difference* between the timelike and spacelike—the centerpiece of all our physical theories—also [in addition to the link between time and informativeness] depends on the system’ (Callender [2017], p. 151). What Baron and Evans ([2021], p. 179) propose is a thesis they call ‘temporal perspectivalism’, where the distinction between time and space is only objective ‘so long as this is understood in a deflationary, epistemic sense’.[7] They admit some invariant structure in the world, existing independently of any perspective, which we then divide into time-like and space-like directions ‘based on our idiosyncratic epistemic constraints and limitations concerning that structure’ (p. 178), so temporal directions are picked out by both the notion of informativeness and the epistemic perspectives of agents, and are not part of the invariant structure. To be clear, they do not refute TIP: they do not contest that the directions of maximal informativeness

[7] Their view is inspired by, and analogous to, the ‘causal perspectivalism’ of Price and Ismael (Price and Corry [2007]; Ismael [2016]). This should not be confused with ‘perspectival realism’ (for example, see Giere [2010]; Teller [2011]; Massimi [2016]).

in our laws pick out time. Instead, the resulting asymmetry between time and space is considered to be a mere artefact of our human perspective, and this view is a consequence of accepting the radical version of TIP.[8]

To flesh this out, as a thought experiment, they consider the possibility of Martian scientists who do not share our epistemic situation within the invariant structure of the world.[9] Imagine that these Martians are 'smeared out' across what we call the temporal dimension and two of our spatial dimensions, and they seek laws that inform along their forth dimension (which would be spatial, for us). They would identify their own natural kinds, as we do, according to the kinds of empirical data available to them, and 'one could imagine complete incommensurability between the two ways of carving up the world' (Baron and Evans [2021], p. 175). Hypothetically, they might design the same PDEs as us, based on the same concerns about informativeness, but would disagree about which directions on the manifold should be regarded as time-like. The point of temporal perspectivalism is that this would be a 'no-fault' disagreement: there is no fact of the matter about who would be correct. Even if communication with these Martians were possible, there would be no clear method for deciding which description to prioritise, and no reason to privilege one over the other in a perspective-independent way. We would have our 'time' as the directions in which our laws are maximally informative; they would have theirs. The 'mere possibility' of such Martian scientists, according to Baron and Evans (p. 175), 'suggests that the distinction [between time and space] is pragmatic in origin'.[10]

Their reasons for adopting this odd-sounding view come from taking the radical reading of Callender's arguments at face value. If time is defined as the direction in which well posed Cauchy problems are maximally informative, then we are at liberty to hypothesize agents for whom this direction turns out to be different from our own. After all, scientific laws are designed very much with our epistemic vantage point in mind—their very purpose is to give information about what we do not have direct empirical access to. For us, this is what we call the temporal future. For the Martians, this might be what we call east or west perhaps. Looking more closely at this thought experiment reveals that it is the orientation of the distinction between time and space that is discussed, not the existence of a distinction in the first place. 'Time' and 'space' are still distinguished for the Martians, only their 'time' is different from ours. We are not invited to imagine the laws that scientists from Venus might come up with if, for example, they had epistemic access to all four dimensions of the manifold. In this case, there would likely be 'no-fault' disagreements with us earthlings about whether there needs to be a privileged time dimension at all, if our

[8] This consequence may be unpalatable to some readers, providing further motivation for looking for ways to reject TIP.

[9] To be clear, this Martian thought experiment is Callender's, but is described very clearly in (Baron and Evans [2021]) and used there to elucidate their position.

[10] Note that pragmatic concerns can come from two sources: our epistemic situation, which is the focus of Baron and Evans, and particular aims and interests, which will be our focus later on. Of course these two sources are related.

concerns lie solely with informativeness. So for Baron and Evans it is taken to follow from Callender's arguments that there are privileged time-directions, but there may be disagreements between agents with differing perspectives about which directions these are. A similar thought experiment, combined with Callender's arguments, could lead to the very definition of time being a matter of perspective.

In what sense are the hypothetical Martian scientists possible? Clearly, their possibility cannot be a symptom of our physical laws because this would induce a circularity in the reasoning. Without an answer to this question, it is difficult to understand how we could motivate anything other than agnosticism about the objectivity of the directions we regard as temporal. Ours might be the only possible epistemic situation for cognitive agents—who knows? It is also difficult to see how any single scientific fact could avoid falling prey to this sort of perspectival reading, since science is necessarily built up from our epistemic perspective and makes use of empirical data we have access to. To avoid these difficulties, we shall explore the extent to which Callender's arguments induce a different sort of perspectival reading. Might there be alternative human perspectives that would disagree about which directions are temporal? If we define time as the direction in which laws are maximally informative, as has been suggested by both Callender and Baron and Evans, this indeed turns out to be the case. This is the case, so long as we do not restrict attention to well posed Cauchy problems. The disagreements between these different human perspectives would be based on their pragmatic concerns relating to the kinds of projects they engage in, since not all scientists specialize in dynamics. This points to an absurdity in defining time in this way; at best, informativeness can be an ingredient in a pluralistic definition of time.

2.3. Temporal aims

We need not consider imaginary alien scientists who disagree about which directions are temporal in order to give a perspectival reading of Callender's thesis. Instead, we shall think about real human scientists whose laws inform in directions that are not regarded as temporal, and others who struggle to generate any accurate information in directions that are uniformly regarded as time by those scientists. TIP is reduced to a particular subset of human interests and pragmatic concerns. Unlike with the thought experiments of Baron and Evans, everyone agrees about which directions to regard as temporal. The 'no-fault' disagreements are about which directions are maximally informative, because this depends on what we wish to be informed about. The connection between time and informativeness is thus more strongly perspectival, because it is as much about the interests and aims of particular groups of scientists as the epistemic situation of humans, and so we replace TIP with TAP: temporal aims proposal.[11]

[11] My analysis of TIP means that it can be thought of as a perspectival truth in Massimi's ([2018b]) sense, because its truth is dependent on a particular set of scientific perspectives (where alternatives are available) as opposed to being dependent on the epistemic situation of an entire species.

There are several ingredients of Callender's argument that can be used to undercut TIP, one of which leads towards TAP. Our focus will be on the commitment to empiricism that, if we recall, is a fundamental ingredient of Humean theories, and Callender's proposal purports to be both Humean and empiricist. This commitment sits uncomfortably with Callender's method of conducting an analysis of the purely formal characteristics of laws as a way of generating metaphysical claims. If we perform a similar analysis whilst holding on to some basic empiricist principles, we arrive at the conclusion that time is not 'the direction *on the manifold* in which we can tell our strongest or most informative stories' (Callender [2017], p. 142; emphasis added), but is instead the direction in which we would most like to be able to tell our best stories.[12] The applicability of the laws to observable, measurable or otherwise empirically accessible physical contexts is all-important here. My view is compatible with that of Baron and Evans but, unlike theirs, does not rely on adopting the so-called radical reading of Callender's proposal; it follows from the 'conservative' reading too. For this reason, it is neither the distinction between time and space in general, nor the particular set of directions that are defined to be time-like, that are relativized to human perspectives. Instead it is only the specific connection between time and informativeness, and this is relativized to some human perspectives.

My main objection is almost preempted by Callender himself. 'Confining attention to the marks of strength but not to strength itself would be a mistake', he warns us (Callender [2017], p. 207). He then goes on to make this very mistake, and the rest of his argument follows from it. His analysis focuses on too narrow a set of laws (identified by the marks of strength they exhibit) to draw a general conclusion about the role of time. He goes on: 'The degree to which a theory is informative is determined by how much of *the world* it manages to imply, not (in the general theory, at least) by formal characteristics' (p. 143; emphasis added). Attributing epistemic priority to the world over formal characteristics of theories is part of the commitment to empiricism, which is central to the BSA and to Humean metaphysics in general.[13] Callender's subsequent analysis of Cauchy problems is in terms of their formal characteristics, therefore only managing to achieve an assessment of their hypothetical informativeness. Their actual informativeness is contingent on these formal characteristics being a good representation of the world. What we shall see is that there are many empirically accessible physical phenomena to which Cauchy problems simply do not apply. Perhaps more worryingly, the directions that are uniformly labelled as time are especially uninformative for many of these phenomena. Section 3 presents two examples to illustrate these points.

[12] The point here is to emphasize not only our pragmatic concerns that stem from our particular epistemic situation, but also those that are about particular interests.

[13] For discussion of these issues, see, for instance, (Carnap [1950]; Quine [1951]; Cottingham [1988]; Lewis [1999]; Gupta [2006]). Feyerabend ([1984]) gives an account of an empiricist epistemological position attributed to Mach's philosophical work, showing how it differs from the use of principles in his physics research.

Ongoing research in the area of dynamical systems theory presented in section 3.3 is focused on developing techniques to try to make better predictions about the temporal future, but often these techniques are complex—thus requiring some relaxation of the other 'horn' of the BSA, namely, simplicity. Nonetheless, one of the central aims of science is to make informative predictions over directions that we have independent (pre-theoretic) reasons to regard as temporal. The connection between time and informative strength in physical laws is thus more plausibly thought of as being entirely pragmatic and not metaphysical, reducible to the aims and interests of scientists rather than to the way the world is.

3. Physical Laws

This section sets out the technical material that is relevant to the arguments presented. In order to fully appreciate the central claims of this article, it is necessary to engage with some mathematical details. Only those equations that contribute towards understanding the philosophical arguments are presented, and their relevance is explained. Section 3.1 describes the structure of PDEs that support well posed Cauchy problems, required to understand the details of Callender's proposal and the 'perspectival' interpretation of Baron and Evans. The mathematical details of two counterexamples to TIP, which instead motivate TAP, are given in sections 3.2 and 3.3. The first of these is a problem for both readings of TIP, while the second is problematic only for the radical interpretation.

3.1. Time and informative strength

Cauchy problems are a kind of boundary value problem (BVP) that can be set for second order linear PDEs in m independent variables.[14] In its most general form

$$\phi(u, x_i, p_i, r_i, s_{ik}) = 0, \qquad (1)$$

for $i = 1, \ldots, m$; $k = 1, \ldots, m$ for $i \neq k$. The solution of the equation, u, is our unknown function of the m independent variables, x_i. The x_is represent physical quantities, and the solution u describes a function relating these to one another. Some arbitrary function of the variables in parentheses is ϕ, where $p_i = \partial u / \partial x_i$, $r_i = \partial^2 u / \partial x_i{}^2$ and $s_{ik} = \partial^2 u / \partial x_i \partial x_k$. The PDE is solved subject to antecedent conditions:

AC_1: $u(x_1, \ldots, x_{m-1}, 0) = u_0(x_1, \ldots, x_{m-1})$.

AC_2: $(\partial u / \partial x_m)(x_1, \ldots, x_{m-1}, 0) = u_1(x_1, \ldots, x_{m-1})$.

[14] My presentation of Cauchy problems and of PDEs is taken primarily from (Garabedian [1964]; Robinson [1998]; Rubinstein and Rubinstein [1998]; Hadamard [2003]; Klainerman [2010]). The simplest Cauchy problem is set for a first order ordinary differential equation, and the definition of a Cauchy problem generalises to PDEs of order n. For ease of exposition, our discussion focuses only on the case of second order PDEs, since these are the most prevalent in physical problems.

Antecedent data of this form are called Cauchy data, defining the problem as a Cauchy problem. Notice that both conditions AC_1 and AC_2 are known functions of $m - 1$ of the m independent variables, and so must be given on a hypersurface of dimension one less than the full solution space.

In order to be 'well posed', a Cauchy problem must satisfy the following three criteria:

WP_1: A solution must exist.

WP_2: The solution must be unique.

WP_3: Solutions must vary continuously with antecedent data.

The first two of these criteria are jointly sufficient to define the equation as deterministic.[15] They ensure that one and only one solution corresponds to each set of antecedent conditions. The third criterion is stronger, requiring that variations of the antecedent data map to corresponding variations of the solution by a continuous function. In practical applications this means that errors made in the computation of either AC_1 or AC_2 are not amplified, or at least that the amplification of errors can be controlled.

In order to establish conditions under which equation 1, together with antecedent conditions AC_1 and AC_2, meets the three criteria to be well posed, leading to the discussion of space and time, we express equation 1 in linear form:

$$\sum_{i,k} A_{ik} \frac{\partial^2 u}{\partial x_i \partial x_k} + \sum_i B_i \frac{\partial u}{\partial x_i} + Cu = f, \tag{2}$$

where A_{ik} is a matrix, B_i is a vector and C is a scalar, all of which we assume to be constant, and f is some linear function of the independent quantities x_i.[16] Equation 2 may be classified according to the eigenvalues of A_{ik} as follows:

Elliptic if and only if all eigenvalues of A_{ik} are non-zero and have the same sign.

Hyperbolic if and only if all eigenvalues of A_{ik} are non-zero, and all but one have the same sign.

Parabolic if and only if any eigenvalues of A_{ik} vanish.

Cauchy conditions AC_1 and AC_2 yield unique solutions only for hyperbolic PDEs. Other kinds of boundary condition (for example Dirichlet or Neumann boundary conditions, which will be defined in sec. 3.2) are either too restrictive for a solution

[15] Note that this is a mathematical definition of a deterministic equation, and should not be confused with philosophical issues about deterministic theories or worlds.

[16] We assume that equation 2 has constant coefficients in the interest of simplicity of presentation. In full generality, the coefficients of a second order linear PDE may be known functions of the x_is, but classification of PDEs with variable coefficients is much more difficult, though not impossible. For the sake of argument, we may assume that the results suitably generalise.

to exist, or are not sufficient to give a unique solution. Conversely, Cauchy data are either too restrictive or insufficient to yield unique solutions for elliptic and parabolic equations. These require either Dirichlet or Neumann boundary conditions. Therefore, a well posed Cauchy problem consists of a hyperbolic PDE subject to conditions AC_1 and AC_2.

The characteristics of a PDE place restrictions on where antecedent data can be defined in order to ensure the existence of a unique solution.[17] Characteristics of equation 2 are defined to be surfaces or hypersurfaces $\xi(x_1, \ldots, x_m) = c$, for constant c, where ξ is a solution of

$$\sum_{i,k} A_{ik} \frac{\partial \xi}{\partial x_i} \frac{\partial \xi}{\partial x_k} = 0. \tag{3}$$

Solving equation 1 for our different classes of PDE shows that elliptic PDEs have no real characteristics, and parabolic PDEs have one family of plane characteristics for each constant x_m. Hyperbolic PDEs, where Cauchy problems are well defined, have two families of conoid characteristics. In $\mathbb{R}^3$ these are pairs of two-dimensional cone-shaped surfaces sharing a vertex at each point of the domain. In higher dimensional spaces, conoid characteristics are higher dimensional generalizations of this basic structure. Figure 1 shows a geometric representation of the characteristics of a hyperbolic PDE in $\mathbb{R}^3$.

The Cauchy–Kowalewski theorem states that to ensure the existence of a unique solution, antecedent data for a PDE must not intersect or be tangent anywhere to a characteristic.[18] Recall that antecedent conditions AC_1 and AC_2 must be defined on a (hyper) surface of dimension one less than the full solution space, and consider where such a surface may be placed so as to avoid the characteristics of a hyperbolic PDE. As shown in figure 2, in $\mathbb{R}^3$ this can be any two-dimensional open surface that passes through the vertices of the cones, whose curvature is such that it never touches the cones. A (hyper) surface on which Cauchy data are defined is known as a Cauchy surface.[19]

A linear hyperbolic PDE that satisfies criteria WP_1 and WP_2, in regions of its domain where solutions are defined for correctly formulated Cauchy data, will also satisfy WP_3.[20] This is the case, provided that solutions do not intersect and are not tangent

[17] Characteristics are important for several other reasons too; for example, in many cases solving the 'characteristic equation' associated with a PDE is a crucial step in solving the PDE itself. For the purposes of this article, however, we only need to be aware of the relationship between characteristics and antecedent conditions.

[18] This was first proved for a special case by Cauchy in 1842 and in full generality by Kowalewski in 1875. The proof requires also that equation 2 be analytic and regular, though these further restrictions are not important for our discussion here.

[19] For the sake of simplicity, this article ignores many of the issues surrounding the lack of generality of the result of the Cauchy–Kowalewski theorem about the relationship between characteristics and antecedent data. Most significantly, there are well-known approaches to general relativity that involve null (characteristic) antecedent data.

[20] Cauchy problems that satisfy WP_1 and WP_2 but violate WP_3 are called ill posed, and are invariably not of the hyperbolic form. They are relatively rare in physical contexts. An example is the inverse gravimetric problem; see, for instance, (Rubinstein and Rubinstein [1998]).

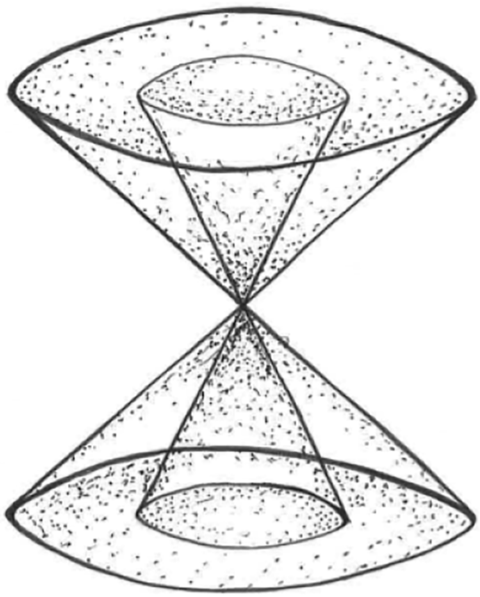

Figure 1. Conoid characteristic surfaces in $\mathbb{R}^3$. The width of the cones is set by the value of the constant c. For a continuous problem there are infinitely many of these surfaces, with a vertex at each point of the manifold.

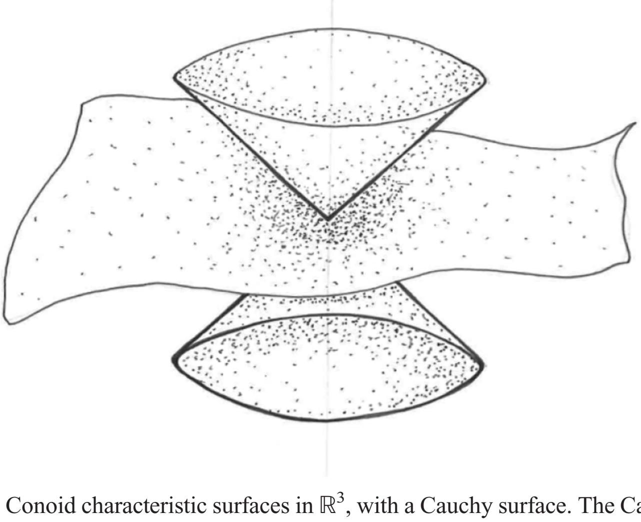

Figure 2. Conoid characteristic surfaces in $\mathbb{R}^3$, with a Cauchy surface. The Cauchy surface extends infinitely in all directions, and the characteristics are defined at every point on the Cauchy surface.

anywhere to a characteristic, and that they do not intersect one another.[21] This means that the solution function to a hyperbolic PDE describes a set of trajectories, each of which passes through a point on the Cauchy surface and propagates inside but not outside the characteristic cone defined at that point.

A canonical example of a hyperbolic equation, which has all of the geometric properties described above, is the wave equation:

$$\sum_{i}^{m-1} \frac{\partial^2 u}{\partial {x_i}^2} - \frac{1}{c^2} \frac{\partial^2 u}{\partial {x_m}^2} = f(x_1, \dots, x_m), \tag{4}$$

for constant c.[22] This equation informs about propagation of electromagnetic or sound waves in directions that we have independent reasons to regard as temporal. We can see that the second derivative of one of the independent variables, x_m, is distinguished by its negative sign. In applications, x_m is chosen to represent time, because this equation is used to describe dynamics, that is, time-evolution.

To summarize the results of this section, we require a clear asymmetry between the hypersurface on which antecedent data are defined and the directions in which solutions propagate, in order to define a well posed Cauchy problem. In physical applications, this means that Cauchy problems provide the most natural means for modelling time evolution, provided the dynamics of the target system are relatively simple and 'well behaved'. For this reason, the Cauchy problem has become known as the Initial Value Problem. So, Cauchy problems are informative in directions that are naturally interpreted as time-like. This fact underwrites Callender's formal argument for his TIP. However, the Cauchy problem is not the only type of problem of physical relevance. This will be emphasized by the examples given in the following two subsections. Their abilities to inform about the manifold of empirical events, considered neutrally, will be compared in section 4.

3.2. Informative, non-temporal directions

This subsection presents a case where laws inform in non-temporal directions. Dirichlet and Neumann problems are two different sorts of BVP that can be set for second order linear PDEs of the form in equation 1 or its linearized form in equation 2. Dirichlet and Neumann boundary conditions are not of the correct form to ensure the existence of unique solutions for hyperbolic equations, but they are for parabolic and elliptic equations. PDEs of the parabolic type, like hyperbolic equations, inform in time-like directions, so we do not discuss them here. Elliptic PDEs, however, do not single out a privileged set of directions, despite providing powerful algorithms that are informative in many areas of physics. For correctly formed

[21] There are less clear results for parabolic PDEs, but these are not the focus of the present work.

[22] Regarding the status of the wave equation (and other PDEs) as laws: the wave equation is derived from Hooke's law, to use conventional terminology. However, what counts as a law and what doesn't is not our concern here.

antecedent data, they satisfy criteria WP_1–WP_3 (despite these being originally explicated in the context of Cauchy problems), thus meeting Callender's standard of informativeness.

A canonical example of an elliptic equation is the Poisson equation:

$$\sum_i \frac{\partial^2 u}{\partial {x_i}^2} = f(x_1, \dots, x_m). \tag{5}$$

This equation has many physical applications, for example, for finding the electric potential for a given charge distribution.[23] The form of this equation makes no distinction between spatial and temporal variables. Recall from the previous subsection that elliptic equations have no real characteristics, so, as long as we are working in real space, the Cauchy–Kowalewski theorem imposes no restrictions on where to define the antecedent data to ensure the existence of a unique solution. The form of the antecedent data, however, is important. As mentioned, Cauchy data are either overly restrictive or lead to instabilities for elliptic PDEs. Instead, we require either Dirichlet or Neumann boundary conditions defined on a closed boundary surrounding the region of interest.

A Dirichlet boundary condition takes the following form:

BC_D: $u(x_1, \dots, x_m) = u_0(x_1, \dots, x_m)$,

for $x_i \in \delta\Omega$, where $\delta\Omega$ is a closed surface or hypersurface. In other words, a Dirichlet boundary condition gives the value of the solution on a closed boundary.

A Neumann boundary condition takes the following form:

BC_N: $v(x_1, \dots, x_m) \frac{\partial u}{\partial x_i}(x_1, \dots, x_m) = u_0(x_1, \dots, x_m)$,

again for $x_i \in \delta\Omega$, where $\delta\Omega$ is a closed (hyper)surface. The function v defines a tangent vector at each point on the boundary. So, a Neumann condition gives the directional derivative of the solution on a closed boundary. Usually, only one of these types of boundary condition is required to solve an elliptic PDE such as the Poisson equation. What elliptic equations do is take information (either the solution or its directional derivative) on a closed boundary and 'evolve' it inwards or outwards from that boundary. An illustration of the two-dimensional case is shown in figure 3. For the three-dimensional case, imagine information on the surface of a soap bubble propagating either outwards or inwards.

Elliptic equations can generate information in multiple directions simultaneously, and there is no special in-built asymmetry between these directions and the boundary on which the antecedent data are defined. Recall that elliptic PDEs have no real characteristics and thus no restrictions on where in the domain this boundary can be, so no particular set of directions is privileged by elliptic PDEs. We therefore have no reason

[23] There are many well-known physical applications for other elliptic PDEs, for example, relating to energy functionals of fields defined over space and time, but the simple example of the Poisson equation is given simply to illustrate the geometric properties of this sort of PDE.

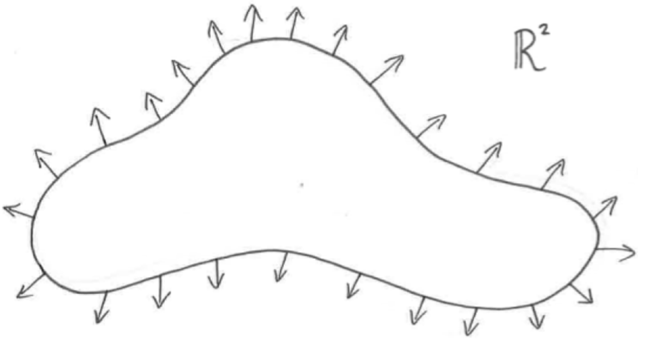

Figure 3. Closed boundary in $\mathbb{R}^2$ where Dirichlet or Neumann boundary conditions may be defined. The elliptic PDE then evolves this information either outwards to fill the manifold exterior to this region, or inwards to fill the interior of the boundary.

to regard the informative directions of an elliptic PDE as temporal. In general these will not correspond to 'time' directions as measured by clocks.

Defining time as the set of directions in which laws generate the most information leads to incompatibilities between the various 'times' emerging from different applications of elliptic PDEs, and between the 'times' arising from analyses of hyperbolic and elliptic PDEs.[24] Callender's reasons for restricting attention to hyperbolic PDEs, rather than giving up on TIP, are discussed in section 4. Next, we come to our second set of counterexamples to TIP, which are not acknowledged by Callender.

3.3. Uninformative, temporal directions

This subsection details a different sort of counterexample to TIP, namely, chaotic dynamical systems, in which the directions that are ordinarily defined to be time are particularly uninformative. This type of example provides motivation for TAP, for the empiricist, because scientists and mathematicians working on the subject of chaos are trying to develop methods for generating information in time-like directions, despite the observed behaviour of the physical world making it especially difficult to do so.

Many physical systems are said to be unstable under perturbations of initial conditions, or chaotic, where initial conditions are now understood to be measured values of our $m - 1$ independent variables, where $x_m = 0$. This is as opposed to any of the antecedent functions u_0 and u_1 described in section 3.1 (AC_1 and AC_2), or either of the u_0s of section 3.2 (BC_D or BC_N).[25] The models now under consideration are

[24] Here, 'times' refers to the set of directions that are maximally informative in each case, the point being that for elliptic equations, there is an absurdity in defining time in this way, since it does not correlate with any other more familiar definitions of time.

[25] Technically speaking, Cauchy, Dirichlet, or Neumann problems whose solutions depend sensitively on the antecedent conditions defined in the previous two subsections would also count as chaotic, but this article aims to focus on the more empirically salient cases concerned with physical measurement.

more directly related to empirical phenomena than the functional equations previously discussed, because they operate at a lower level of abstraction. What is meant by instability here has to do with the way the variables $x_1, \ldots, x_m$ are related according to some function u, which may be (but is not necessarily) the solution to a Cauchy problem. If the measured values of $x_1, \ldots, x_{m-1}$ at $x_m = 0$ are perturbed slightly, giving say $\tilde{x}_1, \ldots, \tilde{x}_{m-1}$, then the values of these variables at $x_m \neq 0$ will also differ from one another, giving rise to two different functions $u = u(x_1, \ldots, x_{m-1}, x_m)$ and $\tilde{u} = u(\tilde{x}_1, \ldots, \tilde{x}_{m-1}, x_m)$. For a dynamical system, these functions will map onto two trajectories in an $m - 1$-dimensional phase space, where each point on a trajectory represents the full state of the system at some value of x_m.

A dynamical system is chaotic if it is both deterministic, in the sense defined in section 3.1, and exhibits sensitive dependence on initial conditions (SDIC). To exhibit SDIC is to be unstable, in the sense that functions u and $\tilde{u}$ will separate at an exponential rate as x_m varies uniformly. For this definition to make sense, we let x_m be identical in both functions, allowing it to naturally represent a time parameter. This means that the distance between phase space trajectories arising from a function, u, will grow exponentially with time. In practical applications, this leads to extreme inaccuracies in predictions of measurement outcomes, since the dynamical equation amplifies errors in the initially measured data. The function, u, therefore does not provide an empirically adequate description of the physical world outside of a very restricted range of values of x_m close to $x_m = 0$. That is, predictions about the future and retrodictions about the past based on these models are practically impossible. In these cases, the asymmetry between the variables $x_1, \ldots, x_{m-1}$ and x_m, corresponding to space and time variables respectively, arises from an asymmetry between what we have empirical access to and what we do not. It is not the case that the direction of increasing x_m is informative; in fact, quite the opposite is the case.

There are many examples of chaotic systems to choose from to illustrate this point. Unlike the linear PDEs we have been analysing so far, which can be grouped into classes that share quantitative features, models of chaotic systems are rather heterogeneous in their mathematical form. What they share is the qualitative feature of a combination of determinism and SDIC.[26] The model presented here is one among many, chosen for its simplicity and physical applicability. Its qualitative features, shared by other chaotic models/systems, mean that the temporal dimension is particularly uninformative.[27]

[26] For a comprehensive analysis of the various types of model that get called 'chaotic', see (Zuchowski [2017]). Her work includes discussion of problems with defining chaos, and with trying to ascertain whether SDIC is a feature of the model or the system. These more involved issues are not our concern here—we assume that SDIC can arise in deterministic models (which are deterministic in the sense defined in sec. 3.1), and that it is a feature of the target system as well as the model because it has empirical consequences.

[27] The philosophical literature on chaos for the most part focuses on modelling problems, especially in 'special sciences', which are tangential to the concerns of the present work; see, for instance, (Werndl [2009]; Frigg et al. [2014]; Zuchowski [2017]).

The Lorenz model is a simplification of a model of convection in fluids.[28] More specific details of this model than what are presented below are largely irrelevant; it must be acknowledged only that the model (and others exhibiting SDIC) was empirically motivated, and solutions can be reproduced experimentally. Other physical systems whose dynamics exhibit SDIC (but which are not modelled by the Lorenz equations) include many problems in celestial mechanics, acoustics, hydrodynamics, some quantum scenarios and even quantum gravity. For classical cases, see, for example, (Berry et al. [1987]; Ott [1993]; Palis and Takens [1993]; Diacu and Holmes [1996]) and for the case of quantum gravity, see (Dittrich et al. [2017]).

This model is a well-known example of a simple chaotic model. It is a system of ordinary differential equations as follows[29]:

$$\frac{dx}{d\tau} = -\sigma x + \sigma y, \tag{6}$$

$$\frac{dy}{d\tau} = -xz + rx - y, \tag{7}$$

$$\frac{dz}{d\tau} = xy - bz, \tag{8}$$

where x is proportional to the intensity of convective motion, y is proportional to the temperature difference between ascending and descending currents, and z is proportional to the deviation of vertical temperature from linearity; σ, r and b are constants; and τ is a time parameter. To connect to our earlier notation, let $x_1 = x, x_2 = y, x_3 = z$ and $x_4 = \tau$. We are now playing a game different from Callender's. Rather than identifying the most informative directions and calling them 'time', we are looking at physical equations with a time parameter already defined in order to investigate its properties. Equations 6–8 each depend on instantaneous values of x, y and z, and solutions give trajectories describing how these variables change over time. Initial conditions for this system are just values of the three variables at $\tau = 0$.

Depending on the values of the constants σ, r and b, and depending on the initial values of x, y and z, solutions of this system can vary dramatically as a result of slight perturbations of initial data. This phenomenon can be quantified by calculating Lyapunov exponents, which measure the rate of divergence between neighbouring trajectories. To spell this out, the solutions $u(x, y, z, \tau)$ and $u(\tilde{x}, \tilde{y}, \tilde{z}, \tau)$ will diverge at a rate of $e^{\lambda\tau}$, where $\lambda > 0$ is known as the Lyapunov exponent.[30] An illustration of this phenomenon is shown in figure 4.

[28] For discussion of the physical applicability of the model, see, for example, (Lücke [1976]).

[29] An ordinary differential equation can be thought of as a special case of a PDE, to connect this to our earlier discussion of PDEs. Many chaotic models do consist of systems of PDEs but these tend to be much more complicated; to present such an example here would distract from our main discussion.

[30] Calculating Lyapunov exponents is just one widely used method for quantifying SDIC that is appropriate in many contexts of physical interest, though not all.

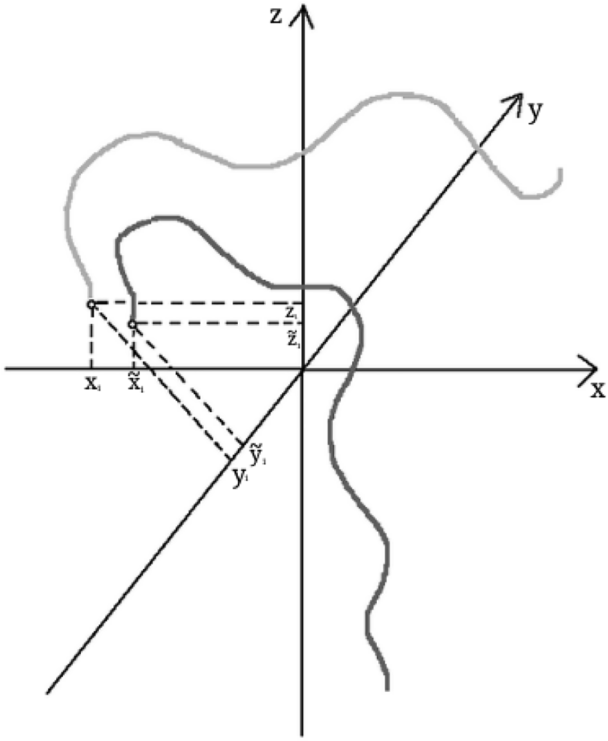

Figure 4. Pair of chaotic trajectories in three-dimensional phase space: $u(x, y, z, 0)$ is depicted in light gray and $u(\tilde{x}, \tilde{y}, \tilde{z}, 0)$ in dark gray. In practice, the two sets of initial values, (x_1, y_1, z_1) and $(\tilde{x}_1, \tilde{y}_1, \tilde{z}_1)$ may be arbitrarily close to one another.

Mathematically, this is a consequence of the nonlinear terms in equations 7 and 8.[31] For this reason, despite the equations having been designed to inform in time-like directions, the information generated can be highly unstable. It may therefore be completely inaccurate with respect to the empirical world because we never have access to arbitrarily precise data through measurement. Approximate predictions about empirical phenomena are generated through statistical analysis of many solutions. This kind of analysis often involves identifying which sets of initial data lead to stable solutions and which do not, rather than simply providing information about what will be measured given one particular set of initial conditions. The empirically adequate information that one can glean from a model of a chaotic system therefore does not correspond to any direction or set of directions on the manifold.

[31] More generally, chaos can arise in deterministic systems as the result of either non-linearity, or discontinuity in the sense of violations of the condition WP_3 defined in section 3.1. This second variety of chaos is much less studied than the non-linear cases, and has been shown only to arise in systems with infinite-dimensionalstate space. See (Kalmar-Nagy and Kiss [2017]) for an example.

The next section explains in more detail why the above cases are counterexamples to TIP, and why they suggest an alternative proposal, TAP.

4. Comparison

This section compares the informativeness (in Callender's sense) of the models described: hyperbolic PDEs are compared with elliptic PDEs in section 4.1, and with chaotic systems in 4.2. In section 4.3, the positive proposal of this article, TAP, is reiterated and compared with the reading of Callender's argument put forward in (Baron and Evans [2021]). Both are forms of perspectivalism, and can be made compatible with one another in the present case.

Callender's argument for TIP relies on his focus on hyperbolic PDEs that support well-posed Cauchy problems. Once we restrict attention to PDEs of this form, as shown in section 3.1, the set of directions that they inform over do indeed correspond to time-like directions in physical applications. It is interesting to observe how their geometric structure enables them to effectively inform over time-like directions, in their domains of applicability. Does this give us any reason to posit a general connection between time and informative strength in physical laws? To answer positively to this question, as Callender does, requires some justification for restricting attention to hyperbolic PDEs, and for taking them to be representative of physical laws in general. Callender justifies this move by appealing to the theoretical virtue of strength as informativeness, inspired by the BSA.

Recall that Callender ([2017], p. 143) stipulates that the 'degree to which a theory is informative is determined by how much of the world it manages to imply'. His claim is then that hyperbolic PDEs with Cauchy data provide laws that are maximally informative in this sense. Is this claim correct? The answer to this question depends on what we consider 'the world' to refer to. Taking our empiricist principles seriously, even in a minimal sense, requires that what we mean by 'the world' should make some contact with empirically accessed or accessible phenomena.

4.1. Hyperbolic versus elliptic PDEs

Callender uses the three criteria (labelled WP_1, WP_2 and WP_3 in sec. 3.1) as a way of making his notion of informativeness precise. Although Hadamard first outlined these conditions in his work on the Cauchy problem (see Hadamard [2003], p. 40), it is not the case that only well posed Cauchy problems meet them. They are sufficient conditions for a Cauchy problem to be 'well posed', but it is not the case that any problem that meets them is a well posed Cauchy problem. A Cauchy problem is defined by the type of antecedent data prescribed for it, and the three conditions were added in order to distinguish those Cauchy problems that are 'well posed' from those that are not.

What Callender has overlooked is the fact that elliptic PDEs such as the Poisson equation, together with the correct sort of antecedent data such as Dirichlet or Neumann boundary conditions, also meet conditions WP_1, WP_2, and WP_3. That is, if such a problem is correctly formulated, there exist unique solutions for each set of antecedent data that varies continuously with those data. What's more, elliptic PDEs in general require only one boundary condition for their solution, in contrast to hyperbolic PDEs which require two. Considered in purely formal terms, then, elliptic PDEs are at least as informative as their hyperbolic cousins. That is, their ability to inform is at least equivalent, provided we make no assumptions regarding the nature of the systems we wish to be informed about.

Dirichlet or Neumann boundary value data are as empirically accessible as Cauchy data.[32] There is a sense in which all three kinds of data are mathematical idealizations, and none of them are directly empirically accessible. Imagine for a moment the practical impossibility of making empirical measurements of some physical quantity on a Cauchy surface. Cauchy surfaces are spatially unbounded and so to collect a complete set of Cauchy data would require an infinite number of measurements. Measurement is also itself a physical process, meaning that data take time to gather and we do not ever have empirical access to every point on a Cauchy surface simultaneously. The idea of an 'initial' function defined on a Cauchy surface, and the stark distinction between time and space built into the definition of this sort of problem, is part of the idealization. In practice, symmetry principles are used so that the initial function can be stipulated or found using an elliptic equation, as mentioned in section 3.2. It makes little sense to compare the availability of initial and boundary data, since both involve a high degree of idealization as well as empirical measurement. It also makes little sense to compare how often each kind of problem appears in physical applications.[33] Both kinds of problem appear in physics, both are used, and both bear some relation to describing and predicting patterns in empirical data.

This point is not entirely ignored by Callender. Although he does not explicitly compare their informativeness, he does briefly argue for the fundamentality of hyperbolic systems, mentioning that 'Geroch (1996) writes, "elliptic and parabolic systems arise in all cases as mere approximations of hyperbolic systems"'. He goes on to say that this point is '*sometimes* demonstrably correct: many equations not of [hyperbolic] form (8.1) can be derived from ones that are by taking limits or making simplifying assumptions. The elliptic Poisson equation, for instance, is a truncation of the linear hyperbolic Maxwell equations' (Callender [2017], p. 165; emphasis added). There arises here the obvious objection that a point that is only sometimes demonstrably correct cannot be a suitable candidate for motivating a general claim, especially given that the 'sometimes' in fact refers only to scalar theories. Gauge

[32] I thank an anonymous referee for raising the issue of comparing the empirical accessibility of the different kinds of data.

[33] It could be claimed, for example, that hyperbolic PDEs are more 'typical' in physical applications. It is not clear how typicality should be judged in the present case: as typicality of use or of physical processes. In either case, there is no measure over the space of PDEs in science, and so this sort of claim would require further evidence if it is to be endorsed. I thank an anonymous referee for raising the issue of typicality.

theories, including general relativity, have constraint equations built in, in the form of elliptic equations, which must be solved in order to set the Cauchy problem.

The above point also relies on taking fundamentality to mean 'non-approximate', when in fact there is no consensus in the literature about what this term means, even in specific contexts.[34] We are then faced with at least two options for how to read 'non-approximate'. Callender's analysis of the above quotation from Geroch suggests a formal reading, where approximation is a particular kind of mathematical derivation. This reading is antagonistic to our empiricist principles, because no mention is made of fit to empirical data. Hyperbolic systems are not more exact (accurate or precise) than elliptic systems in the sense of being closer approximations to what is measured in the physical world.[35] Callender's reasoning is thus not sufficient to favour the Maxwell equations over the Poisson equation, or to prefer hyperbolic over elliptic systems in general. No argument is given for regarding hyperbolic systems as more informative than elliptic systems. Neither can be considered in isolation as the best guide to investigating the role played by time in physical laws. They are simply used in different ways, depending on the kind of information sought.

When comparing the informativeness of hyperbolic and elliptic systems, it is important to be aware of what it is we wish to be informed about. That is, our aims must be acknowledged. If we wish to be informed about dynamics, an elliptic system will not do. Given that the study of dynamics just is the study of processes evolving in time, it is to be expected that the mathematical tools designed for this purpose distinguish time-like directions, as hyperbolic PDEs do.

4.2. Stable versus chaotic dynamical systems

Chaos is studied as a branch of dynamical systems theory, where the aim is to inform in temporal directions. Chaotic dynamical systems are characterized by the fact that this task is made particularly difficult by the behaviour of such systems. The point is not to dwell on the metaphysics of laws, but simply to apply the principles of the BSA. This means taking some set of physical phenomena and looking for the best (strongest and simplest) methods for describing the observed regularities and for making empirically relevant predictions.

As emphasized in section 3.3, the study of chaotic systems has empirical motivations. The goal is to develop the most effective methods for making empirical predictions in time-like directions, a procedure that often involves a variety of techniques to mitigate the instability. In general it is the case with these systems that reasonably accurate predictions can be made only for short timescales, even when using our best

[34] It is not the intention here to engage in discussion about what exactly is meant by 'fundamental'. The interested reader may refer to, for example, (Tahko [2018]) and references therein, and especially (Crowther [2019]) for the case of physical theories. The issue is mentioned here only for the sake of completeness, and readers may have in mind some basic intuitive notions of fundamentality.

[35] This point also applies to chaotic systems, and will be returned to in section 4.2.

predictive tools. Our best description of such a system should surely take this into account. So, if we seek the best description of some physical system exhibiting chaotic behaviour, we see quite the opposite of Callender's TIP. That is, the temporal directions about which we wish to be informed are especially uninformative.

The informativeness of hyperbolic PDE systems depends on the behaviour of the target system—a hyperbolic system does not always faithfully encode the observed dynamics. Temporal directions in well posed Cauchy problems have the potential to be especially informative due to the way the problems have been designed. However, they inform only about possible (non-actual) empirical phenomena unless the dynamics of the target system are 'well behaved' in the sense of being non-chaotic. In other cases, alternative methods (usually statistical) are used to try to make the best possible empirical predictions, but it does not make sense to speak of 'the direction(s) of information propagation' when such methods are employed. The dynamical equations generate information over time, but this information is wildly inaccurate when compared to empirical phenomena. The dynamical equations for such systems are also not at all simple, usually being characterized by non-linearity. Statistical analysis of phase portraits is more informative in the sense of giving accurate information, but the information generated is not temporally oriented. The non-temporally oriented statistical methods also keep track of the regularities of chaotic systems, but these are generally 'higher level' or 'emergent' regularities.[36] The point is not exactly to decide which formal methods count as laws, either 'properly speaking' or according to the BSA, but to understand how we come to be informed about physical systems and the role time plays.

Callender ([2017], p. 145) recognizes that 'there is no guarantee that the best theory will contain algorithms that permit such a nice time', but claims that 'nature must be kind. So far, it has been'. Chaotic phenomena show that this claim is false: they are cases where nature is not 'kind', where the best theories consist of amalgamations of statistical methods that do not contain algorithms permitting a 'nice time'. Information in temporal directions is what is sought, but accurate and precise information (in the sense of correctly corresponding to empirical data) is not what is found. The existence of systems exhibiting such behaviour in the physical world, and recognition of the great efforts made by mathematical physicists to generate at least approximate short-term predictions in such cases, together motivate replacing TIP with TAP.

Objections to my argument from chaos may again appeal to the supposed fundamentality of hyperbolic systems, as well as the non-fundamentality of chaotic systems. This objection would provide an alternative reason for favouring hyperbolic over chaotic systems, despite their informativeness about the empirical world being

[36] This is mentioned in relation to a point raised by an anonymous referee, who questioned whether the formal methods presented count as laws. Laws, for the Humean, simply keep track of empirical regularities. For the best systems theorist, they do so in the 'best' way. If the 'best' way of describing a chaotic system is not strong or simple enough to satisfy certain best systems theorists, they must admit that there are empirical phenomena that exhibit some regularity but about which there are no laws.

incomparable, thus supporting a weaker version of TIP on the grounds that the informative directions in our most fundamental laws are time-like. Taking 'fundamental' to mean either 'non-approximate' or 'exact', so long as we are empiricist and exactness has something to do with fit to empirical data, means that it is simply not the case that hyperbolic systems are more fundamental than chaotic ones.

If chaos were to arise only in complex systems where approximations to the underlying dynamics are made, the objection to TAP from fundamentality, where 'fundamental' roughly means 'exact' or 'non-approximate', might carry some force. However, there is no entailment in either direction between chaos and complexity, and chaos often arises in systems that are not complex.[37] The approximate statistical methods mentioned in section 3.3 are used in the study of chaotic dynamical systems to deal with the SDIC that is present in the more precise description. SDIC does not arise as a result of approximation. This refutes the possible objection to the significance of chaotic systems based on their non-fundamentality, where 'fundamental' is taken to mean 'non-approximate'.

If 'fundamental' were instead taken to refer to that which is described by putatively fundamental theories, such as general relativity, quantum field theory or quantum gravity, the objection that chaotic dynamics are non-fundamental still does not hold. As maintained in (Dittrich et al. [2017], p. 554), 'full general relativity is almost certainly chaotic'. The same authors then go on to make the case that quantum gravity involves chaotic dynamics. Of course these claims are controversial. The idea that general relativity should involve chaos in the form of non-linear equations has met with some resistance, perhaps based largely on aesthetic concerns and stipulations of physical reasonableness. However, as Wheeler ([1964]) pointed out, 'it is no objection to the physical reasonableness of general relativity to find that the equations are non-linear. To argue that physics "does not like" non-linear equations is as futile as standing under the roar of Niagara Falls and trying to reason away the hard reality of non-linear hydrodynamics'. Regarding the quantum case, it is an established fact that chaotic dynamics occur in quantum systems,[38] although here the situation is not nearly as straightforward as in the classical case described in section 3.3.

The discussion of fundamentality is somewhat tangential to our main discussion. Empiricist principles commit us to take seriously the available empirical data, and this alone does not involve prioritizing certain sets of data over others. The issue of fundamentality is mentioned only to alleviate the potential concerns of those empiricists who are also reductionists, who believe we should not be looking to 'special sciences' to address metaphysical issues. Chaos is not only a 'special science' phenomenon, and so it should be taken into consideration in a thorough investigation into the role played by time in physical theories. What this phenomenon demonstrates is a failure of our ability to predict, despite scientists' best efforts. This failure is enforced by

[37] For discussion of where chaos and complexity come apart, see, for instance, (Ladyman et al. [2013]).
[38] For comprehensive treatments of the subject, with reference to experimental results, see (Haake [1991]; Stöckmann [2000]).

the dynamical behaviour of the physical world. It shows that it is neither physics in general, nor the behaviour of the physical world, that grants time its informative strength in those special cases where temporal directions in laws successfully inform. Rather, it is the way in which scientists construct laws, to provide the kind of information that is sought, which is very often information about unknown, empirically inaccessible temporal directions.

4.3. Two forms of perspectivalism

The arguments given in this article have been in support of the claim, referred to as TAP, that the connection Callender makes between time and informative strength in physical laws holds only relative to a particular set of aims and research interests. This was demonstrated by the counterexamples presented. TAP can be viewed as a perspectival reading of one of the central claims of Callender's proposal, which I have been referring to as TIP. The 'perspective' to which it is reduced is understood to mean some collection of research perspectives. It is suggested that physics which aims to make predictions in directions that are already understood to be 'time' will have 'time' as the informative direction in its laws. When such laws do not generate accurate and precise information due to the observed behaviour of the systems to which they are applied, as is the case for chaotic systems, alternative techniques are used to gain as much information as possible in 'time' directions.

The proposal put forward in (Baron and Evans [2021]) is a different sort of perspectival reading, where 'perspective' is understood as the epistemic perspectives of some group of agents, in this case the human species, or 'creatures like us'. Their 'temporal perspectivalism' is explicitly based on the 'causal perspectivalism' of Price ([2005]). To explain what is meant by a perspectival claim, Price uses the example of foreigners, reminding his readers that who is regarded as 'foreign' depends on perspective. The people who are 'foreign' to Frenchmen are different from those who are 'foreign' to Englishmen, but this does not mean that either party is incorrect about their judgements. The same kind of analogy can be applied to the ways in which physical laws are used to generate information. There will be 'no-fault' disagreements about which directions are informative, depending on what kind of information is sought. There will also be potential 'no-fault' disagreements about which directions are informative depending on the epistemic perspective, a hypothesis that can be fleshed out more thoroughly by identifying what it is about a particular epistemic perspective that makes it suitable for gathering information along a particular set of directions. The former, interest-relative, perspectivalism is what has been referred to here as TAP. The latter is the species-relative perspectivalism developed in (Baron and Evans [2021]), here applied to 'informative directions' rather than 'time directions' as in the original work. The two forms of perspectivalism are compatible, when applied to the same claim, since epistemic perspectives and research perspectives identified by aims and interests are not independent.

5. Conclusion

Callender's claim (TIP) is that our most informative models inform in time-like directions. This article has proposed a modification to this claim (TAP), which says instead that what is the most informative set of directions in the most informative model depends on what we wish to be informed about, that is, on our aims. The extent to which models geometrically distinguish time from space depends on the extent to which we wish to be informed about processes evolving in time (dynamics), where we have a host of independent reasons to regard this evolution as happening in time. This point is highlighted by consideration of elliptic systems, where the aim is to find information in spatial directions and not to study dynamics.

The extent to which the 'time' directions in a model, designed with the aim of informing over time, are in fact informative with respect to a physical system of interest depends on the behaviour of that system. This is demonstrated by the case of chaotic systems, where the aim is to study dynamics but the 'time' directions in the models are uninformative. The empiricist principles Callender set out with are particularly important with respect to this last point, because they force us to think about informativeness in terms of correspondence of a theory or law with empirical data. The proposal, TAP, should be understood as a fairly weak claim: scientists often (but not always) wish to generate information in temporal directions. Where the aim is to generate such information, and where the physical world behaves in such a way as to allow for such information to be generated (both are required), strong algorithms may be applied that, unsurprisingly, inform in temporal directions.

These messages illustrate the more general problem that the actual achievements or capabilities of science can sometimes fall short of its aims. Where some scientific discipline sets out to make accurate and precise empirical predictions, there always exists the possibility of failure. In the case of chaotic systems, it remains to be seen whether this is only a failure of our current best models to achieve the predictive aims, or an in principle incapability of any model to make the kinds of predictions sought. The way that the subject has been presented here suggests the latter interpretation, although this will most likely depend on the particular system in question. Future developments in the field may also turn out to demonstrate otherwise.

To return very briefly to the question of a philosophical underpinning for the various asymmetries between time and space, Callender's account cannot succeed. This article has pointed out the lack of generality of the key component of his account, namely, the connection between time and informative strength in physical laws, and suggested a perspectival reading of it. Recognizing that this claim cannot underpin the asymmetries between time and space, its metaphysical thrust consists in a relationship between the world and information-gathering systems such as ourselves. This article has pointed out that this relation only sometimes obtains, but that often it is the objective of scientific inquiry to forge such a relation. Identifying when this relation obtains and when it doesn't, and asking why it does or does not obtain in these cases, would be interesting projects for deepening the analysis.

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